THE FORCE OF THE WIND

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PREFACE

In the course of my researches on the subject of aviation I have frequently noticed the absence of any work dealing with the subject of wind pressure in relation to engineering, and I am inclined to think that this little book meets a want that has long been felt by engineers.

The formulæ are as simple in each case as circumstances and moderate accuracy will permit, and calculus methods are only introduced in three places, where it seems almost impossible to avoid them.

I have endeavoured throughout to bring the matter as nearly up to date as possible, and have included the results recently published by Eiffel and Lanchester. By excluding all matters not *immediately* concerned with wind force or of those which have not yet attained engineering

importance, the work has been reduced to very small dimensions.

I take this opportunity of thinking Mr. Galbraith for assistance on the subject of Stresses in Masonry.

HERBERT CHATLEY.

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CHAPTER I

PRACTICAL IMPORTANCE OF WIND PRESSURE

THE practical problems in which wind pressure plays an important part are so numerous that a complete list would be of cyclopædic dimensions, but a classification may be made on the following lines:

1. *Wind Load on Structures.*
2. *Resistance to Moving Bodies.*
3. *Power Production.*

The first group will include all cases of wind pressure on roofs, bridges, and buildings.

The second will comprise air resistance to trains, projectiles, ships, and flying-machines.

The third covers the cases of windmills and sailing-vessels.

A yet more general classification can be obtained by grouping the second and third types together in one,

describable as pertaining to the combined effect of wind and independent motion, so that the two classes become

1. *Pressure on Stationary Bodies.*
2. *Pressure on Moving Bodies.*

It is scarcely necessary to point out that very rarely is the wind negligible in any structural or mechanical design. Thus in the case of a large railway bridge, destruction (implying a large value for the static equivalent load) may be produced by the wind, and in locomotion the air resistance may be quite comparable with track or gradient resistance.

In regard to power production, the windmill and the sailing-vessel both owe their propulsive effort to the wind pressure on aerodynamic surfaces, and a very considerable technical skill is required to design and manipulate these appliances so as to employ with a maximum efficiency the kinetic energy of the wind when the latter varies in both magnitude and direction.

In no subject is the importance of **relative motion** more marked than in this. The main principle involved is well illustrated in the following example.

If a bird be flying through the air so that *from the earth* he is making an average velocity of 20 miles per

hour in a northern direction, and the wind is observed to be blowing from the north to the south with an average velocity of 10 miles per hour (*i.e., as compared to the earth*), then the bird is moving through the air with an average velocity of $20 + 10 = 30$ miles per hour, or we may also say, the air is moving past the bird (regarding the latter as fixed) with the velocity of 30 miles per hour.

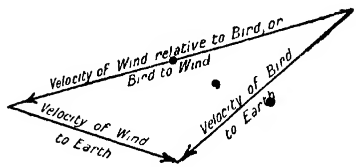


FIG. 1.

If the directions are not in the same line, the diagonal of a parallelogram, whose sides are the two components, gives the relative velocity (Fig. 1).

We may then say *the resultant or velocity of any body relative to the earth is produced by combining the velocity of the wind, also relative to the earth, with the velocity of the body relative to the wind, or of the wind to the body.*

By the use of this principle all problems of the second class become reducible to those of the first, so that the general problem of wind pressure is this: Given the relative velocity (including "direction") of

the wind to any body, to find the forces produced by the reaction which occurs as the wind is deflected from the body.

Hence in all problems concerning the force of the wind, two things are required:

(1) The relative velocity before impact;
(2) The relative velocity after impact.

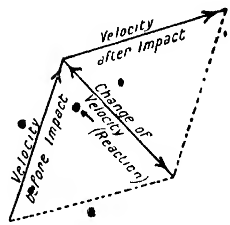


Fig. 2.

The first is determined by the conditions of the case. The second depends on the form and nature of the surface in regard to its action on the first. Herein lies the difficulty in most problems, and usually assumptions have to be made which are not wholly warranted.

When, however, the two are known, the "vector difference" (*i.e.*, the quantity which, together with the first velocity, produces the second velocity) indicates the change of velocity. This multiplied by the mass

of air concerned per unit time (generally "lbs. of mass per second") gives the force acting. Its direction is the same as that of the "vector difference" (Fig. 2).

A further difficulty here arises as to the point of application of the force ("centre of pressure"). These points will be discussed and illustrations of the process outlined above will be given in the following pages.

CHAPTER II

IMPULSIVE FORCE OF THE WIND

It has already been pointed out that the pressure exerted by air having a velocity relative to any body is dependent on the manner in which that velocity is altered by the impact, the force being the time-rate of change of the momentum. If the velocities were, in each case, simply uniform in magnitude and direction there would be no difficulty in computing the force produced; but it generally happens that both the incident and reflected streams of air are very complex in form, and the velocities of the different parts are very varied.

In chap. iv. the general principles of the "Stream Line Theory" will be given. At present it is sufficient to say that a "stream line" is the locus of a particle travelling past the object on which force is exerted. Imagining every one of the particles of air to have its path mapped out by such lines, it is easy to see that a knowledge of the general trend of such lines would

afford a means of knowing the momentum before and after impact (Fig. 3).

Before, however, describing the stream system for any specific case, it will be useful to classify the different elements which affect the question:

(1) The density of the air.

(2) The form of the obstacle.

(3) The actual magnitude of the velocity of approach.

(4) The smoothness of the surface of the obstacle.

(5) The rigidity of the obstacle.

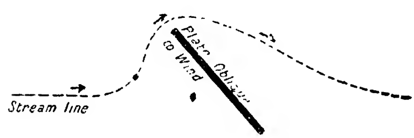


FIG. 3.

(1) The density of the air (about 0·08 lb. per cubic foot) varies so little that there is no necessity to refer particularly to this at present.

(2) The form of the obstacle is probably the most important element of all. If a section be taken of the obstacle in the plane of a stream, the manner in which the stream is diverted by it is all-important in determining the reaction.

(3) Upon the actual velocity of approach much depends. At very small velocities the resistance varies as the velocity; at most small velocities (up to say 100 miles per hour) the resistance varies as the square of the velocity, and at very high velocities the resistance varies up to the fifth or sixth powers reaching a maximum variation when the speed of sound in air (1100 feet per second, or 750 miles per hour) is reached. It then decreases in variation again, and finally only varies as the square of the velocity.

(4) The smoothness of the surface affects the resistance in a great degree. A very irregular or rough surface produces a frictional resistance which varies with the irregularity and the area of the surface.

(5) If the obstacle be not firmly placed, or if the relative velocity of the air change rapidly, impulsive forces of great magnitude may be produced.

We may broadly say that the force depends, in any instance, on four factors.

(*a*) *Head Resistance* due to primary diversion of the incident streams.

(*b*) *Negative Pressure* due to the formation of a partial vacuum at the rear of the obstacle.

(*c*) *Eddymaking force*. waves of eddies or vortices form behind the obstacle.

(*d*) *Skin Friction*, referred to in (4) above. (Fig. 4.)

The literature and experimental work on this subject is enormous, but particular mention may be made of the names of Hutton, Duchemin, Dines, Langley, Maxim and Eiffel. Work on high velocities has been done by Bashford and Mayevski. On the mathematical side the names of Lord Rayleigh, Professor Greenhill, Professor Bryan and Mr. Mallock will show that the subject has by no means been neglected.

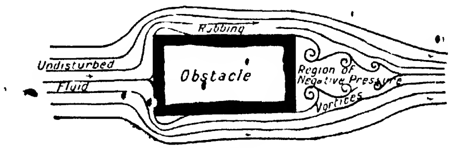


FIG. 1.

The general character of the results obtained is as follows:

(*a*) *Head Resistance* is found to vary with (for instance) some power of the velocity, so that we may write

$$R_h = kV^n$$

As already mentioned the value of n varies from 1 to 5. For the usually occurring velocities n may be taken as 2.

(*b*) *Suction or negative pressure* is found generally to bear a ratio of about 0·6 to 1·0 to the head resistance.

This includes, however, the very variable item (c), wave-making force, which, as Mr. Mallock has shown, depends chiefly on the difference between the relative velocity and the velocity of propagation of sound.

(d) *Skin friction* at ordinary speeds is found to follow roughly the law (for unit area)

$$R_s = kV.^2$$

All the items vary directly as some power of the area not greatly differing from unity, so that a general formula may be written

$$AR = A(R_h + R_n + R_s) = A[1 \cdot 6 k V^n + kV^2]$$

It has been a general custom to abbreviate this into a simple formula.

$$R = KAV^2$$

It is obvious that in some cases this rule will be very inaccurate, but it is nevertheless true in the majority of practical cases. Ballistics should be mentioned as a special exception.

The constant K will have a different value for each type of obstacle.

The following types of obstacle may be specially mentioned.

(α) Square thin plate perpendicular to the mean direction of the relative wind.

(β) Oblong ditto.

(γ) Circular ditto.

(δ) Irregular ditto.

(ϵ) All the above (α, β, γ, δ) inclined at any angle θ to the mean direction of the relative wind.

(ζ) Any combination of plates such as above.

(η) Perforated plate.

(ι) Rectangular solid, axially placed.

(κ) Cylindrical " "

(λ) Sphere " "

(μ) Ellipsoid " "

(ν) Cylindrical solid with any diameter in direction of wind.

(o) Irregular solid in any position.

(π) Solid of least resistance.

(ρ) Ship and Ichthyoid forms.

(σ) Hollow obstacles (*e.g.*, open sheds).

(τ) Tubes.

It is an easy matter to arrive at the maximum value for K on a flat-faced obstacle, if we assume the truth of the law KAV^2 where A is the area projected on to a plane perpendicular to the direction of approach, by assuming that all the momentum is lost by the air. The quantity of air that approaches per second is AV, and its momentum is

$$\frac{.08}{32.2} \cdot AV^2 = \text{say } 0.0023\ AV^2$$

Actually the loss of momentum is never so great as this, so that we may at once say that K is less than 0·0023. On the assumption of the force as the space rate of change of kinetic energy we get K just half this value and in all practical cases this is increased by negative pressure, so that

$$K = \left(1 + \frac{1}{n}\right) \times \cdot 0012$$

where n is greater than unity. Many experimenters make n about 3, so that

$$K \text{ becomes } = \cdot 0016$$

It should be understood that this value especially applies to case (a) in the above list.

In cases β, γ, δ, *i.e.*, plane areas other than square in form, normal to the direction of flow have slightly different values for K. It is found that K increases slightly with the area if the shape remains the same; and also that K increases with the perimeter, the area remaining the same. The circle has, therefore, the least value for K, being about 1 per cent less than the value for the square area. Rectangles having a side to side ratio of 1 to 2 shew an increase in K of about 1 per cent., and with a side to side ratio of 1 to 4, an increase of 2 per cent. above the square. An increase of area in the ratio of 8 : 1 shows an increase of K in

the ratio of about 10 : 1, so that the law would perhaps be better expressed in the form

$$K A^m V^2$$

where m is about 1·1, and K has the constant value ·0016. Seeing, however, that this does not take into account the variation of perimeter when area is constant, and also that the index of the velocity is not

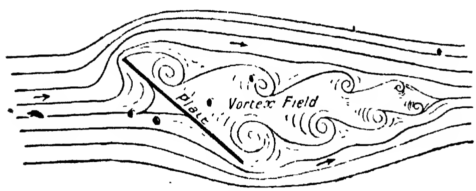


FIG. 5.

absolutely constant, the simpler form is perhaps preferable. In any case the variation is not very great and an equal error may easily be made in estimating V.* As regards an irregular area it should be remembered that the resistance increases with the perimeter. Exactly to what extent is not known, but Mr. Lanchester's research on peripheral flow should be consulted by those who wish to obtain a detailed knowledge of the information possessed on this point.

Case c has probably received more attention than

* See paper in *Engineer*, April 17, 1908.

any other, and a complete account of the work done would fill many volumes (Fig. 5).

The simple rule adopted from Lord Rayleigh is quite satisfactory for most practical cases and is as follows:

$$P_n = P_{90} \times 2 \sin \theta$$

where θ is the angle between the plate and the wind, P_n is the normal pressure and P_{90} is the pressure on the plate when square on as above discussed. A maximum pressure is reached at about 30° inclination and thereafter (*i.e.*, from 30° to 90°) the variation is small and uncertain.

M. Eiffel gives a very simple rule

$$P_n = P_{90}\left(\frac{\theta^\circ}{30}\right) \text{ for values of } \theta \text{ from } 0^\circ \text{ to } 30^\circ$$

and $P_n = P_{90}$ from 30° to 90°. It is certain, however, that the perimeter is of considerable importance. A narrow plane with its long edge towards the wind experiences a greater thrust than with its narrow edge in that direction, the difference amounting to some 20% above and below the values given above, which refers to a square plate.

The next case, combined plates, is of great importance in bridge work, and the results are at first sight very paradoxical. The simplest combination conceivable is that of two plates, both normal to the wind, separated

by a certain distance. If this distance is considerable (say more than five times the least width of the plates) the resistance of the two is about twice that of a single plate. If, on the other hand, the distance apart bears only a small ratio to the least width of the plates, then the joint resistance is considerably less than that of the two plates considered as independent units. Thus if the ratio is unity (*e.g.*, for circular plates a diameter apart) the total resistance is about one-half the resistance of a single plate. If the ratio is one-half (*e.g.*, two circular plates a radius apart) the resistance is about three-quarters that of a single plate.* As the distance increases the joint resistance increases, so that at a distance ratio of 3 the joint resistance is about 1·6 the single plate resistance, and at a ratio of 4, 1·8 times (*Baker*).

Experiments disagree on the resistance when the plates are close, Thibault and Eiffel making it smaller than the single plate, Sir Benjamin Baker making it in no case smaller. Eiffel's recent results were obtained by a very elaborate and careful process, so that they are probably correct, the diminution being explicable on the grounds of a neutralisation of the pressure on the leeward plate by the negative pressure on the windward plate (Fig. 6).

Eiffel.

If the plates are both inclined to the wind, the difference is less noticeable. Thus two plates inclined at an angle θ less than 30°, and having the distance ratio more than unity, have a joint resistance about twice that of a single plate, interference only recurring at smaller angles.

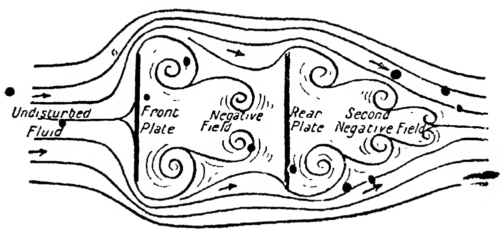


FIG. 6.

We may generally conclude that in structures, where the distance of any two plates, however arranged, is more than *twice* the least dimension of the windward plate, they may be considered independently. The case of a perforated plate has not received very much attention. As has been pointed out by Professor Claxton Fidler,* it is quite erroneous to take the net area of the grating as the effective resistance considering it as a flat plate. In passing through the perforations, the streams contract so that there is almost the same

* *Treatise on Bridge Construction.*

discharge through the plate as would take place through the space were the plate absent. This being so, there must be a high velocity through the orifices and great friction and eddying. Mr. Gaudard's formula:

$$P = p(S - 0{\cdot}65\sigma)$$

where P is the resistance, p the ordinary pressure per square foot on a normal plate, S the over-all area of the plate, and σ the aggregate area of the openings.*

Passing next to the consideration of solids (cases l to p) we notice first the somewhat unexpected result that the resistance of a solid is generally less than that of a plate having the same frontal aspect.

This is accounted for by the decrease in the negative pressure and eddying. The stream tubes diverted by this incidence on to the face return to the sides of the solid, run along them and finally meet again in the rear after a comparatively small interval. If we can guide the stream lines so that they continuously hug the sides of the solid, we have a form of "least resistance" (see next chapter).

As an example of the quantitative results the following comparisons will be useful.

Square plate normal to stream 1·0.

Cube (same linear dimensions) 0·80.

* Eiffel gets an increase in ratio of resistance = 22 for two lattice works one depth apart.

Rectangular prism (length = 2 diameters)		0·72.
„	„ 3 diameters	0·71.
„	„ 4 diameters	0·72.

(*Du Buat.*)

The effect of length in diminishing the eddying is here very apparent. After 3 diameters the frictional resistance on the surface makes a small increase in the resistance.

A similar set of results from M. Eiffel's recent experiments will be useful.

Disc normal to stream	0·071
Cylinder 1 radius long	0·071
„ 1 diameter long	0·069
„ $1\frac{1}{2}$ „ „	0·051

The case of a cylinder transverse to the wind is of considerable importance in chimney and similar problems. Francis Wood, M.I.C.E., in his *Practical Sanitary Engineering*, gives the following ratios:

Square solid, transverse to wind	1·0
Hexagonal	·75
Octagonal	·65
Circular	·50

By hydrodynamical theory* the resistance to a cylinder

* See Lamb's *Hydrodynamics*, p. 87.

is zero, but this is only true in the absence of friction and under the critical velocity at which discontinuity takes place (see next chapter). Similarly with any profile with continuous curvature.

Cases o, π, ρ can only be studied in the light of the stream line theory, which is the subject of chap. iv.

The case of an open shed is of great practical import. A structure open towards the wind and completely closed on the windward side is subject to a very considerable thrust. If the area normal to the wind be A, then the resistance is upwards of

$$2 \times 0{\cdot}0016\, A\, V^2,$$

because the forward momentum of the wind has to be entirely converted to a backward one in order that the air shall escape. If the trend of the air is an upward one in leaving the building, the latter tends to be lifted bodily, the magnitude of the lifting force being determinable by taking the vector change of velocity, squaring it and multiplying by 0·0016 times the area in square feet of the opening to wind.

In the case of the entrance to a tube, the resistance depends on the change of velocity at the mouth. It is shown by writers on hydraulics that this is proportional to $(V-v)^2$ where V is the velocity before

entrance and v the velocity after entrance, and that the loss of energy per lb.

$$= \left(1 - \frac{1}{r}\right) . \frac{V^2}{2g}$$

where r is the ratio of the outer area to the tube sectional area, and V the velocity of approach. This

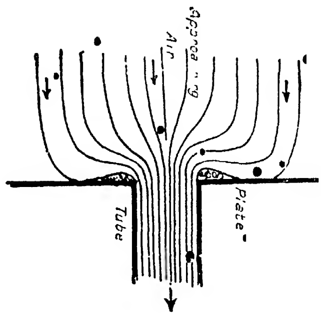


FIG. 7.

may be converted by the usual rule connecting pressure energy and kinetic energy, *i.e.*,

$$p = \frac{wV^2}{2g} = \cdot 0012\, A\, V^2$$

(notice that eddying is here neglected and may be the cause of a greater resistance).

w here stands for the weight (lbs. per second) discharged, and p is pressure (lbs. per square foot), A the

area of the windward part of the plate in which the opening is.

Hence we have resistance due to orifice

$$= \frac{\cdot 0012 \, A V^2}{\left(1 - \frac{1}{r}\right)}$$

This does not, of course, include the *resistance of the plate itself.* (Fig. 7.)

It will, of course, be understood that at each individual point of any obstacle the direction of the pressure is practically *normal* to the surface.

CHAPTER III

VARIATIONS IN VELOCITY

An important, but comparatively little considered, question is the variation of velocity in the wind.

The velocity changes with the following factors:

(1) Place (*i.e.*, on the earth: latitude and longitude)
(2) Height (above sea level).
(3) Temperature.
(4) Barometric pressure.
(5) Time (short periodic changes: "gusts").
(6) Physical topography.

Its changes may, of course, be in magnitude or direction, such changes being of long or short period.

A rapid change is generally known as a "gust." A change of long period is termed "a change of wind." The actual structure of the wind is very complex, but there is a general trend to or from certain centres of minimum or maximum pressures, frictional and dynamic resistance due to obstacles causing local eddy motion.

Owing to the decrease in the viscous resistance, density and pressure at heights, the velocity increases

with elevation. The exact law is not known, but generally it may be assumed that the ratio of the velocities is about equal to the fourth root of the ratio of the heights concerned. There are generally winds of regular magnitude and direction at a considerable height, but these will not affect engineering practice. As regards places it is well known that the maximum velocities attained by the wind are vastly greater in the tropics than in temperate latitudes, although occasionally in the latter a velocity of 100 miles per hour is observed.

The first and most important consideration is the variation in height. To find the centre of pressure on an exposed surface whose lower edge is H_0 above the sea level, at which a maximum velocity V_0 may be assumed, we proceed as follows:

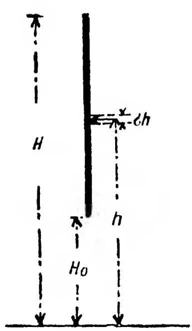


FIG. 8.

$$P \propto V^2 \qquad (1)$$

$$\frac{V_1}{V_2} = \sqrt[4]{\frac{h_1}{h_2}} \qquad (2)$$

so that

$$\frac{V}{V_0} = \sqrt[4]{\frac{h}{H_0}}$$

$$V^4 = \frac{h V_0^4}{H_0}$$

$$p = K\sqrt{\frac{h V_0^4}{H_0}} = \frac{K V_0^2}{\sqrt{H_0}} \cdot \sqrt{h}$$

$$dp = p\,b\,.\,dh = \frac{K\,V_0^2 b}{\sqrt{H_0}}\,.\sqrt{h}\,.\,dh = \beta\,.\sqrt{h}\,.\,dh$$

$$\frac{\Sigma\,(dp\,.\,h)}{\Sigma\,(dp)} = \bar{h} = \frac{\displaystyle\int_{H_0}^{H} h\,.\,dp}{\displaystyle\int_{H_0}^{H} dp.}$$

$$\frac{\displaystyle\int_{H_0}^{H} h^{\frac{3}{2}}\,.\,dh}{\displaystyle\int_{H_0}^{H} h^{\frac{1}{2}}\,.\,dh} = \frac{\left(3\,(H^{\frac{5}{2}} - H_0^{\frac{5}{2}}\right)}{\left(5\,(H^{\frac{3}{2}} - H_0^{\frac{3}{2}}\right)} \qquad (3)$$

If $\qquad H_0 = 0$, then $\bar{h} = \frac{3}{5}\,H \qquad$ (3A)

and in any other case

$$\bar{h} > \frac{3}{5}\,H, \text{ becoming } = H \text{ when } H_0 = H$$

Rule 3 A will be found useful in designing chimneys or walls exposed to pressure.

The total pressure is of course

$$\frac{2}{5}\,.\,\frac{K\,V_0^2 b}{\sqrt{H_0}}\,.\left(H^{\frac{3}{2}} - H_0^{\frac{3}{2}}\right) \qquad (4)$$

The exact relation between temperature and barometric pressure are very uncertain. Of course, in accordance with the law of convection, air flows upwards from places of high temperature and descends when cooled; also, it will flow from places of high barometic pressure to those of low, the general flow being approximately normal to the isobars or line of equal pressure.

Some general notion of the flow of air under a difference of temperature or pressure may be obtained from the following rules which depend on the conservation of energy:

$$\Delta p = \frac{wV^2}{2g}$$

where $\Delta p =$ difference of barometric pressure at two places, $\frac{w}{2g} = \cdot 0012 =$ half-mass per cubic foot at mean temperature, $V =$ velocity, feet per second:

$$V = \sqrt{\frac{2g \cdot \Delta p}{w}} = \sqrt{\frac{\Delta p}{\cdot 0012}} \qquad (5)$$

$$J\kappa t = \frac{V^2}{2g}$$

where $\kappa = 0{\cdot}1721 =$ specific heat (at constant volume) of air (units of heat to raise 1 lb. 1 degree Fahrenheit), $J =$ conversion constant (heat units to work units)

= 778, t = difference of temperature at two places, so that

$$V = \sqrt{2g \cdot J \cdot \kappa \cdot t} \qquad (6)$$

With regard to the effect of gusts, precise information as to the duration and range of gusts is wanting, but many valuable data will be found in Professor Langley's *Internal Work of the Wind* (Smithsonian Institution, Washington).

It is pointed out by Fidler (*Bridge Construction*) that if a wind blowing with a steady pressure p is suddenly augmented to the pressure $p + \Delta p$, the stress will be proportional to $p + 2.\Delta p$.

This, however, is only true if the pressure is instantaneously applied, so he suggests the following formula:

$$P = \max. p + \eta (\max. p - p).$$

where P is the effective pressure, p the mean pressure, and η a coefficient less than unity.

Transforming this into terms of velocity we have:

$$KV_E^2 = KV^2_{max} + \eta K (V^2_{max} - V^2) \qquad (7)$$

where κ is the usual coefficient, V_E is the effective velocity, V_{max} the maximum velocity, and V the mean velocity.

To render this rule of practical use it is, of course

necessary to know the maximum and mean velocity of V, and assume a value for η.

According to the Forth Bridge anemometer readings the average steady wind pressure was about 36 per cent., the maximum of small anemometer plate readings. Taking the ratio of pressures as the square root of the ratio of the velocities, the latter amounts to 60 per cent., so that

$$V_1^2 = \frac{25}{9} V^2 + \eta \left(\frac{25}{9} V^2 - V^2 \right)$$
$$= \frac{25}{9} V^2 + \eta \left(\frac{16}{9} V^2 \right)$$

$$\text{If } \eta = \tfrac{1}{2} \text{ then } V_1^2 = \frac{11}{3} \, . \, V^2 \qquad (8)$$

This is an extremely high result and must not be taken to imply a *real pressure* 11/3 as great as the mean steady pressure, but as the *static pressure* corresponding to the dynamic effect of a suddenly acting gust.

According to the Bidston Observatory records, only about once in eight years is a pressure of close on 40 lbs. per square foot attained.

The recent work at the National Physical Laboratory is noteworthy in regard to the question of irregularity in the structure of the wind.

Numerous experiments carried out there have failed

to show any very great difference in the mean pressure per square foot on a small structure and on a similar but much larger one, so that the data given above will apply without dangerous inaccuracy to structures of any size.*

In considering questions of wind pressure, the local conformation of land and existing structures should, of course, always be noticed. Thus, a long street with its length in the direction of the prevailing winds will be subject to a steady current of high velocity, the effect on a structure at the end of such a street being probably greater than if the structure were wholly exposed. Or, again, a structure may be so sheltered that little or no wind need be allowed for.

* See papers by Dr Stanton in the *Proc. Inst. C.E.*, from vol. clvi. to date.

CHAPTER IV

STREAM LINE THEORY

MENTION has several times been made of stream "lines" and "tubes," and it is now necessary to give more attention to this subject. Especially is this seen to be the case when it is realised that the resistance in all cases is not a simple question of loss of momentum or kinetic energy.

A stream line is an imaginary line which maps out the path of a particle of moving fluid. Strictly speaking, the term should be restricted to the case where the path is, owing to the constancy of the conditions, invariable in direction, the word "line of flow" being used when a less rigid system is considered. If we imagine that a mass of air of considerable depth and breadth is travelling in a given direction, the lines of flow will be parallel lines drawn in that direction when the velocity of each particle is equal and in the same direction as that of every other particle. It is convenient, although not wholly true, to regard an

unresisted wind as of this character. The stream lines are infinite in number, but, for convenience, may be drawn any suitable distance apart. If we imagine a number of such lines enclosing a cylindrical space, then, since the particles in that space have lines which do not pass out of the cylinder, the system is termed a "tube" or "filament." Under steady conditions it is, of course, impossible for two stream lines to intersect, as this would imply that successive particles could cross each other's paths without colliding.

If now the assumption is made that we have a perfect fluid, the quantity flowing along a stream tube will not lose any energy. It is customary to call the energy per lb. the "head." This head may be of three kinds:

(1) Potential.
(2) Pressure.
(3) Kinetic.

By potential head we mean the distance in feet which the fluid can yet descend under gravity. By pressure head we mean the height in feet corresponding to the pressure exerted by the fluid.

Since pressure in a fluid is proportional to the density and the depth, so that we may write pressure (lbs. per sq. foot) = weight (lbs. per cubic foot) × depth (feet)

the depths in feet ("pressure head") corresponding to the pressure:

$$= \frac{\text{pressure, lbs. per sq. foot}}{\text{weight, lbs. per cubic foot.}}$$

By kinetic head we mean the height in feet corresponding to the velocity possessed by the fluid.

Since any body falling freely by gravity gets a velocity

$$V = \sqrt{64 \cdot 4 \times \text{drop in feet}}, \text{ the height } = \frac{V^2}{64 \cdot 4}$$

The total energy therefore per pound is constant and equal to

$$h + \frac{p}{w} + \frac{V^2}{2g}$$

where h is the potential head or drop in feet,

$\frac{p}{w}$ is the drop in feet corresponding to the pressure,

$\frac{V^2}{2g}$ is the drop in feet corresponding to the velocity.

If, now, the tube is always filled with fluid the quantity discharged per second will always be the same, so that this quantity passes each point in the tube per second. If we multiply the cross-section of the tube by

the velocity the result is the quantity, so that at any point the

$$\text{velocity (feet per sec.)} = \frac{\text{quantity (c. ft. per section)}}{\text{area of section (square ft.)}}$$

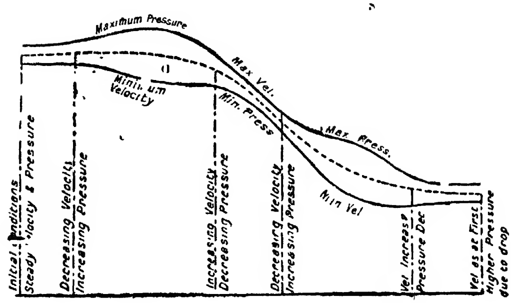


Fig. 9.

Hence we may deduce the following conclusions:

(1) If a stream tube descends towards the earth, the pressure or velocity, or both must increase, since h is decreasing.

(2) If the tube enlarges the velocity decreases and the pressure increases.

(3) If the tube contracts the velocity increases and the pressure decreases.

Having thus considered the fluid in the tube, we must next consider the effect of bending the tube into any form.

If the tube is doubled back so as to form a "U," then the total reaction to pressure in the direction of the bend (*i.e.*, parallel to the arms and from the direction of arrival of the fluid)

$$= \frac{2 \times \text{weight per sec.} \times v}{32{\cdot}2}$$

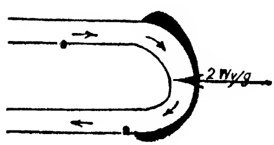


FIG. 10

for, since force is the time rate of change of momentum and the momentum *per second*

$$\text{which is } \frac{\text{weight per sec.} \times v}{32{\cdot}2}$$

is turned completely from a forward direction to a backward direction, the total change of momentum per second is $\frac{2\,w\,v.}{g}$

If the tube is bent to an angle less than this last (180°) the change of momentum must be found by a triangle of velocities as shown in chap. i.

If the tube is bent in any manner, so that the one end is eventually parallel to and in the same direction

as the other, the total force on the tube is zero (Fig. 11).

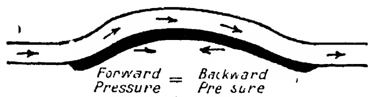


FIG. 11

This last case is the most important of all in finding forms of least resistance, for, if a stream tube can be made contiguous with an obstacle, and also the direction of departure be the same as that of arrival, the pressure on the obstacle is nil (Fig. 12).

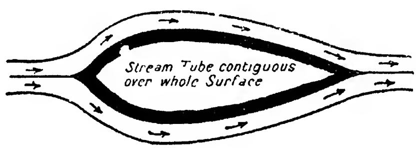


FIG 12.

Hence in the absence of friction we may say that if the stream tubes adjacent to the obstacle are diverted from their rectilinear paths the force acting upon the obstacle is found by considering (1) whether the stream lines return to the original direction immediately after passing the obstacle; and (2) whether the stream lines coincide with the profile of the obstacle throughout its length.

It will be understood that at some distance past the obstacle the stream lines necessarily return to their original direction, the loss of momentum possibly experienced at the obstacle being made up from the surrounding air. If, however, the stream line leaves the surface of the obstacle before having returned to its original direction, the pressures due to the flexure of

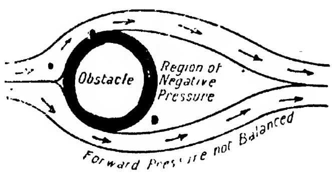


FIG. 13.

the stream tube will not be balanced *on the obstacle*, so that a "dynamic" resistance is experienced (Fig. 13). When the stream line leaves the obstacle before having passed it, it is said to describe a "line of discontinuity." The space between the obstacle and this line is filled in an actual fluid (at a pressure less than normal) with vortices, which contract as they are swept away from the obstacle, finally vanishing at a certain point in the rear of the obstacle where the lines of discontinuity from each edge of the obstacle meet. A certain force is, of course, required to generate these vortices.

In an actual fluid there is also a certain resistance

due to viscosity or "skin friction" between the obstacle and the stream tube passing over it and contiguous with it. Small eddies, acting as it were as rollers for the body of the fluid, form a layer over the surface and so produce a certain resistance. Including small, but unavoidable "dynamic" resistances, even a form in which the stream lines are apparently contiguous throughout experiences a considerable resistance, about $\frac{1}{10}$. KAV^2 where K is ·0016 and A the central section.

In the light of this theory it is possible to arrange surfaces so as to get a minimum resistance. Various forms may be termed "of least resistance," the conditions being that the profile shall so divert the stream lines that they do not leave the same, and that they shall be guided by the after parts, so that the lines meet immediately behind the object without any discontinuous flow.

These conditions are approximately realised by all smooth and easily rounded objects, but the nearest approach to the minimum resistance is obtained by the use of a wedge directed towards the stream, expanded with an easy cissoid or sine curve to the largest section, and contracting again with a curve having no reflex curvature to a fine taper at the rear. The body "lines" of ships are excellent examples of this type (*see* Fig. 12).

For further information concerning stream lines the following may be consulted:

Rankine, *Shipbuilding*.

Froude, Papers in *Transactions of Inst. Naval Archs.*, *&c.*

Hele-Shaw, Papers in *Transactions of Inst. Naval Archs.* and *British Association*.

Greenhill, "Hydromechanics" in *Encyclop. Britt.*

Lamb, *Hydrodynamics*.

Basset, *Hydrodynamics and Sound*.

CHAPTER V

STRESS IN STRUCTURES DUE TO WIND

As has already been indicated, the effect of wind is to produce an additional load having a lateral component, the total force at each point being normal to the exposed surface. On sloping surfaces, such as roofs, the pressure will be largely vertical and will have a maximum intensity on slopes greater than 30 degrees, closely approaching the intensity on a vertical surface, say 50 lbs. per square foot. In view of the proximity of other structures, which will to some extent screen the roof from wind and diminish the velocity of the wind, it is not usual to take greater pressures than 40 lbs. per square foot. On vertical surfaces and open bridgework the higher value should be used.

We may for convenience classify structures subject to wind pressure into two groups:

(1) With wind surface.

(2) With frame only.

The first class will include roofs, walls, and the like. The second refers to open bridge-work and framing. It is customary in the first class to assume that such surface bears uniformly on all its supports, and that the wind pressure is uniform and over the entire surface. That this condition is not actually realised is quite

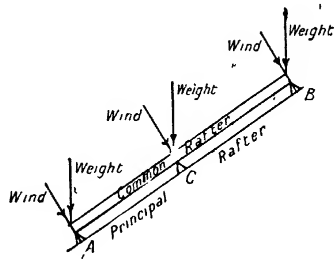


FIG. 11.

certain, but the practical departure from the same does not seem to be important.

In this manner we obtain a number of forces acting on the supports of the surface, which may be considered as loads normally acting. The stresses so produced may be analysed separately, or a combined diagram produced by adding (with a parallelogram of forces) the wind and the static load at the same place.

Thus, if the illustration indicates a sloping surface, *AB*, which is 20 feet long and supported in the direc-

tion perpendicular to the paper at every 12 ft., an area of 240 square feet needs support on each truss. If the

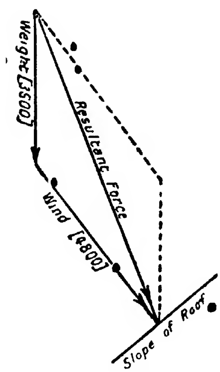


FIG. 15

actual weight per square foot be 30 lbs. and the wind be taken at 40 lbs. per square foot we have

$30 \times \frac{1}{4} \times 240 = 1800$ lbs. dead load at A and B.
$30 \times \frac{1}{2} \times 240 = 3600$ lbs. " " C.
$40 \times \frac{1}{4} \times 240 = 2400$ lbs. wind load at A and B.
$40 \times \frac{1}{2} \times 240 = 4800$ lbs. " " C.

The actual load at C will be found by drawing the diagonal of a parallelogram whose adjacent sides measure 3600 lbs. and 4800 lbs. respectively, the first

vertical and the latter inclined from the *vertical* at the same angle the slope makes with the horizontal.

Considering this resultant at each joint on the windward side and the ordinary dead loads at all others we find the reactions by the usual methods, so ascertaining the combined stress in the various members.

If, as is usually the case, both sides of the building are liable to be subjected to wind pressure, the combined stresses, found as above, must be considered in designing both sides of the frame.

It is not unusual to treat the wind-stresses by the second method, viz., as independent forces. The following examples of a roof will illustrate the process:

The conditions of equilibrium are, of course,

$$\Sigma\,(p) = \Sigma\,(\mathrm{R}),\ i.e.,\ \mathrm{P} = \mathrm{R}_1 + \mathrm{R}_2$$

where P = total load and R_1, R_2 are the reactions

$$\mathrm{R}_2\mathrm{C} = \frac{Pa}{2} + \frac{Pb}{4}$$

where C is the span, a and b are the distances of $\frac{P}{2}$ and $\frac{P}{4}$ from the left support all perpendicular to the forces. If the roof has a roller on one side the reactions will not, of course, be parallel to the wind, and must be found by link polygon in the usual manner.

Another example of wind surface occurs in the steel cage method of constructing buildings. If the frame

consists of rectangular bays a great bending effect may be produced at the joints, and even more important is

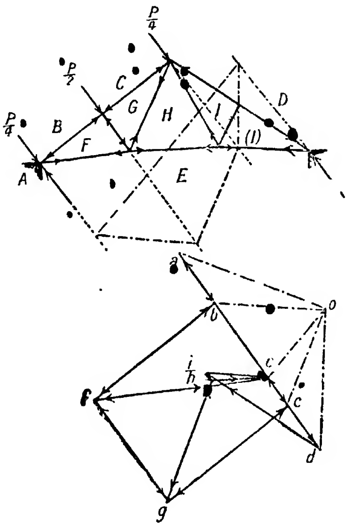


FIG 16.

the shearing effect on the whole frame, which tends to distort the rectangles into rhomboids.

If a frame of the type shown is subject to a wind pressure on the left side having a total magnitude P, the height of the centre of pressure being Ph, the frame at the attachment to the ground is subject to a bending

moment Ph and a shearing force P, so that each vertical is subject to an axial push or pull $= \frac{Ph}{d}$ and a shearing force $= \frac{P}{2}$.

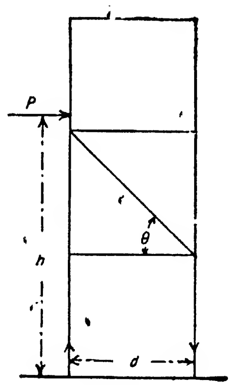


FIG 17.

If we regard the total pressure up to any point h as $= \beta\, h^{\frac{1}{2}}$ (see formulæ 1 and 2 in chap. iii.) we have the shearing force h_1 from the ground

$$= \mathrm{P} - \int_0^{h_1} p \cdot dh$$

$$= \mathrm{P} - \frac{2}{3}\beta h_1^{\frac{3}{2}} \qquad (1)$$

and the bending moment at any point, h,

$$= \int_{h_1}^{H} ph \, . \, dh$$

where H is the total height.

Since $p = \beta h^{\frac{1}{2}}$, we have

$$M = \beta \int_{h_1}^{H} h^{\frac{3}{2}} \, dh = \frac{2}{5} \beta \left[H^{\frac{5}{2}} - h_1^{\frac{5}{2}} \right] \qquad (2)$$

In all these expressions $\beta = \dfrac{K V_0^2}{H_0}$ as given in chap. iii.

The tensile force in the windward column, or the compressive force in the leeward column, may be found by equating the bending moment (2) to the quantity Td, where T is the required force and d the distance between the columns.

Thus in the case illustrated at a point h_1 above the ground, the breadth of a single bay being b, the tension or compression in the columns is

$$T = \frac{2 \beta b \left[H^{\frac{5}{2}} - h_1^{\frac{5}{2}} \right]}{5d} \qquad (3)$$

In the horizontal beams there will be a compression equal to the shearing force, and if, as is usually the case, diagonal wind-braces are used inclined at an angle

θ, the direct tension or compression in these will be S sec θ and

$$S = \frac{b(P - \frac{2}{3}\beta h_1^{\frac{3}{2}})}{\cos\theta} \qquad (4)$$

Details of construction will be found in Freitag's *Architectural Engineering* (Wiley, New York).

In the case of a structure, such as a wall or chimney, in which for constructional reasons the elevation tapers, the centre of pressure is not in reality so high as given in chap. iii., but it will nevertheless be convenient to disregard this correction, so that the total turning moment may be taken as

$$= \text{total pressure} \times \frac{3}{5} H =$$

$$R = \frac{2}{5}\beta b (H^{\frac{5}{2}}) \times \frac{3}{5} H = \frac{6}{25}\beta b H^{\frac{7}{2}} \qquad (5)$$

[Note that H_0 is, except in evaluating β, disregarded in most cases.]

Equating this to the moment of resistance we have

$$\frac{6}{25}\beta b H^{\frac{7}{2}} = \frac{f I}{y}$$

so that the bending stresses (f. lbs. per square foot)

$$= \frac{6\beta b H^{\frac{7}{2}} y}{25 I} \qquad (6)$$

where y = distance from neutral axis of bending to the extreme (windward or leeward) edge of the base section and I is the moment of inertia of the section about the same axis. All measurements are in lbs. and feet.

These bending stresses will be combined with the dead-load compression on the base section, so that the maximum compression, when W = total weight, A = area of base section,

$$= \frac{W}{A} + \frac{6\,\beta\,b\,H\,y}{25\,I} \qquad (7).$$

and the minimum compression

$$= \frac{W}{A} - \frac{6\,\beta\,b\,H^2 y}{25\,I} \qquad (7\text{A})$$

It is a common condition in *masonry* structures that there shall be no tension, *i.e.*,

$$\frac{W}{A} \nless \frac{6\,\beta\,b\,H^2 y}{25\,I} \qquad (8)$$

Writing

$$\frac{W}{A} = \frac{6\,\beta\,b\,H^2 y}{25\,I}$$

we get conditions which limit the weight or area as follow :

$$W = \frac{6\,A\,\beta\,b\,H^2 y}{25\,I} = \frac{6\,\beta\,b\,H^2 y}{25\,k^2} \qquad (9)$$

where k is the radius of gyration of the section about the afore-mentioned axis of bending, and

$$A = \frac{25\,I\,W}{6\,\beta\,b\,H^{\frac{5}{2}}\,y} \qquad (10)$$

Thus in the case of a hollow circular section, internal radius r, external radius R.

$$\begin{aligned} A &= \pi\,(R^2 - r^2) \\ I &= \pi\,(R^4 - r^4) \div 4 \\ y &= R \end{aligned}$$

$$W = \frac{24\,\beta\,b\,H^{\frac{5}{2}}R}{25\,(R^2 + r^2)} \qquad (9\text{A})$$

$$R = \frac{24}{50}\cdot\frac{\beta\,b\,H^{\frac{5}{2}}}{W} \pm \sqrt{\frac{24}{50}\,\frac{\beta\,b\,H^{\frac{5}{2}}}{W} - r^2} \qquad (10\text{A})$$

This formula is in error in the following respects:

(1) b is assumed uniform up the height, whereas it actually tapers. Taking b as the mean breadth will approximately correct this. (See next correction.)

(2) The roundness of the face is neglected. Taking $\frac{b}{2}$ instead of b will correct this.

A further difficulty occurs in connection with the fact that b will generally be related to R, the external base radius. It will therefore be necessary to work out this problem by a series of trial cases converging to the best practical solution.

Passing now to the consideration of open-framed structures, of which the most important practical example is bridge-work, it will be found that a few simple rules serve to explain the method of dealing with such structures.

On each member of the frame there will be, during hurricanes, a pressure depending in the form of the member and the manner in which the same is presented to the wind, the value of which may be computed from the data given in chap. ii. This pressure will produce stresses in the members directly and indirectly, *i.e.*, the individual member will be subject to bending moment, and the whole frame will be affected by the reactions at each end of the member. The latter will need to be considered as a further load usually horizontally applied, particular attention being paid to the turning effect produced in the foundations of the bridge. As regards the individual members it will be necessary to apply the usual methods of dealing with laterally loaded struts and ties.*

[NOTE.—Railway bridges must be considered as covered by carriages in calculating wind surface.]

In the case of a strut fixed at both ends it is shown in the text-book referred to that the maximum bending

* See Perry's *Applied Mechanics*, p. 170.

moment produced by a uniformly distributed lateral load w per foot run, and an end thrust F, is

$$= \tfrac{1}{4}\, W l\, \frac{E I \pi^2}{4 l^2} \Big/ \left(\frac{E I \pi^2}{4 l^2} - F \right) \qquad (11)$$

where W is the total lateral load (wl lbs.), l the length of the strut (feet), E the modulus of elasticity (lbs. per square foot) and I the moment of inertia about the neutral axis of bending.

The further relation,

$$\left(1 - \frac{p}{f}\right)\left(1 - \frac{p}{\beta}\right) = \frac{W l}{4 f Z} \qquad (12)$$

is important as fixing f, the maximum compression stress. Here

$$p = \frac{F}{\text{area of section}} = \frac{F}{A}$$

$$\beta = \frac{E I \pi^2}{4 l^2 A}$$

$$Z = \text{modulus of section } \frac{I}{y}$$

y = distance from neutral axis of bending to edge of section.

In the case of a tie we take

$$\frac{F}{A} \pm \frac{M}{Z} = \left\{ \begin{matrix} \text{maximum or} \\ \text{minimum} \end{matrix} \right\} \text{tensile stress.}$$

where F is, as before, the axial force (tension in this case).

A is the sectional area,

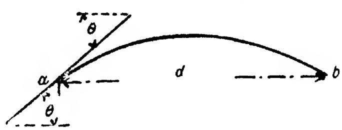
M is the bending moment due to the lateral force

$$= \frac{w\,l^2}{8} = \frac{W\,l}{8}$$

Z is modulus of section, as above.*

The stresses in a sheet of material subject to wind pressure may be computed by the usual rules for plates and shells under uniformly distributed loading. These are too complex to give here, but may be found in Perry's *Applied Mechanics*, and most engineering note-books. One case only may be considered.

The stress in a spherical segment subject to internal wind pressure may be computed as follows.



Let *ab* be the section of the segment which is fixed at the edge. The total pressure transverse to the diameter *ab*

$$= \frac{\pi d^2}{p4}$$

If there is a circumferential tension *f*, the total

* See also paper in *Phil. Mag.*, March 1892.

magnitude of this is $\pi d t f$, where t is the thickness. The transverse component of this is $\pi d t f \sin\theta$, so that

$$p \frac{\pi d^2}{4} = \pi\, dt . f \sin\theta$$

$$\frac{pd}{4} = t . f . \sin\theta .$$

$$f = \frac{pd}{4\, t \sin\theta}$$

CHAPTER VI

WINDMILLS

THIS is the first subject in which wind force has to be considered in relation to the production of power. Windmills or wind-motors are of very numerous types, but may be divided broadly into two classes.

(1) Large diameter wheels with a few (generally four) wooden or cloth blades.

(2) Small diameter wheels with numerous steel blades.

The manner of constructing wooden mills is well described in Weisbach's *Mechanics of Engineering* (volume on "Hydraulics") and I only propose to consider the principles according to which power is obtained from the wind.

It is customary to confuse wind wheels with propellers. Although in some respects the one is the converse of the other, yet two very important points compel us to treat them differently.

(1) There is no motion of the wheel in the direction of the wind.

(2) Owing to a continuous and previously undisturbed supply of air meeting the wheels (except above a certain high critical speed) there is no question of the blades diminishing each other's action.

It will be convenient to make two assumptions as to the flow of the air.

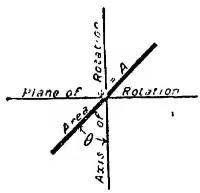


FIG. 18.

(1) Before impact the wind is blowing axially to the wheel. This condition is almost exactly secured by the steering wheel or vane.

(2) After impact the air flows along the surface of the blades and leaves it relatively in the same plane.

Neither of these assumptions can be absolutely true, but we are almost obliged to make them and the actual discrepancy does not seem to be very great.

If the blade be plane and, at the instant of striking, stationary, the pressure in it is easily found by the rules given in chap. iii.

Normal pressure=

$$N = \cdot 0016\ A\ V^2 \times \frac{\theta^\circ}{30^\circ}$$

The component of this pressure in the direction of the axis is N sin θ, and this fall produces pressure in a step bearing, energy being wasted by friction there.

The component in the plane of rotation is N cos θ, and this produces rotation if, when there are m blades, m N cos θ exceeds the resistance of the blade due to friction and load. It is necessary to express this in the form of torque to be useful.

If the blades, as is usually the case, taper towards the centre, the breadth at any point can be expressed as λr where λ is a constant, and the area of any elemental strip of the blade is $\lambda r \, . \, dr$, so that we can write

Turning moment on one blade =

$$\cdot 0016\ \lambda\ V^2 \cos\theta \int_{r_1}^{r_2} r^2 \, . \, dr$$

where r_1 and r_2 are the radii of the internal and external bounding lines of the blades; hence for the whole wheel the torque

$$T = \frac{\cdot 0016\, m\, \lambda\, V^2 \cos\theta\, (r_2^{\,3} - r_1^{\,3})}{3} \qquad (1)$$

If this exceeds the total resisting torque M, the wheel will commence to revolve so that

$$\ddot{\theta} = \frac{T - M}{I} \qquad (2)$$

where $\ddot{\theta}$=angular acceleration (radians per sec. per sec.)

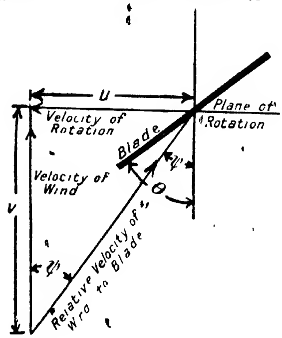


FIG. 19.

and I = moment of inertia of whole wheel about axis of revolution.

As the wheel rotates the angular acceleration will decrease on account of air resistance and slip until the velocity is constant, its actual value depending on the velocity of the wind (assuming the resistance of wheel and mill constant.)

To study the wheel under working conditions it is therefore necessary to consider the reaction when the blade is moving.

If u is the velocity of the blade in the plane of rotation, the actual direction in which the wind is moving relatively to the blade is found by drawing a triangle with u and V for its sides, the diagonal giving the relative velocity.

The angle of impact is

$$\theta - \psi \text{ where } \psi = \tan^{-1}\left(\frac{u}{V}\right)$$

so that the normal pressure

$$N_1 = \cdot 0016\ A\ V_o^2 \frac{(\theta - \psi)^\circ}{30^\circ} \text{ where } V_o = \sqrt{u^2 + v^2}.$$

and its component in the direction of rotation is

$$N_1 \cos\theta = \frac{\cdot 0016\ A\ V_o^2\ (\theta - \psi)\ \cos\theta}{30} \qquad (3)$$

and the work done per sec. is

$$N_1 \cos\theta\ .\ u\ . = \frac{\cdot 0016\ A\ V^2\ (\theta - \psi)^\circ \cos\theta\ u}{30^\circ} \qquad (4)$$

The volume of air approaching the blade is

$$A \sin(\theta - \psi)\ .\ \ V_o$$

and its kinetic energy

$$= \cdot 0012\, v^2\ .\ V_o\ .\ A\sin(\theta - \psi)$$

so that the efficiency is, approximately,

$$\frac{\cdot 0016\ A\ V_o^2\ (\theta - \psi)^\circ \cdot \cos\theta}{\cdot 0012 \times 30 \times v^2\ .\ V_o\ .\ A\sin(\theta - \psi)}$$

If we write $30^\circ = \frac{\pi}{6}$, and let $(\theta - \psi)$ in radian measure $= \text{Sin}\,(\theta - \psi)$, this simplifies to

$$\frac{2{\cdot}75\, V_0 \cos\theta\, u}{v^2} \qquad (5)$$

This expression is at the best only an approximate one

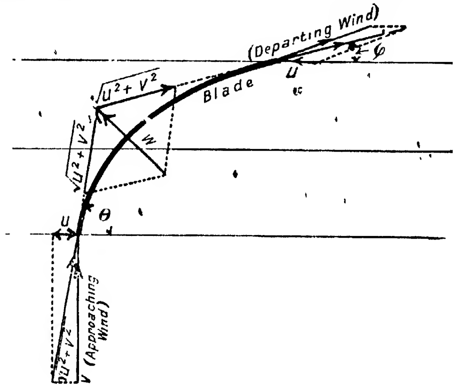


FIG 20.

for an actual mill, since $V_0^2 = u^2 + v^2$ where u is a quantity depending on the radius of the wheel.

Before stating the simpler rules used, we will consider the case of a wheel with curved blades, such as are now extensively made in steel. If the diagram shows such a blade moving laterally with a velocity u and subject to a wind moving axially, the relative motion of the wind is inclined at an angle $\tan^{-1}\frac{u}{v}$ to the axis. If

$\theta = \tan^{-1}\left(\frac{v}{u}\right)$ there will be no turbulence at entry, and the air will flow easily along the blade with a velocity $= \sqrt{u^2 + v^2}$.

If it leaves with this velocity in a direction ϕ, *i.e.*, parallel to the blade, its absolute motion (as compared to the earth) will be compounded of this relative motion and the absolute motion of the wheel u, as shown by the upper parallelogram of velocities.

The alteration of velocity in passing round the curved blade is easily found by drawing a third parallelogram of velocities (the central one in the figure), in which the two sides of the parallelogram are each equal to $\sqrt{u^2 + v^2}$, and the diagonal is the change of velocity. It is obvious that this is greatest when θ is a maximum $= \tan^{-1}\left(\frac{v}{u}\right)$, and $\phi = o$, *i.e.*, the blade is tangential to the plane of rotation at the rear. The direction of change of velocity is $\frac{\pi}{2} - \left(\frac{\theta + \phi}{2}\right)$ inclined to the plane of rotation, and its magnitude

$$w = \frac{\sin(\theta - \phi)\sqrt{u^2 + v^2}}{\cos\left(\frac{\theta - \phi}{2}\right)} \qquad (6)$$

If the weight of air concerned is W the total change

of momentum $= \frac{W}{g} \cdot w$. This is the total pressure, and resolving it into components normal and parallel to the plane of rotation we find that the force producing rotation is

$$\frac{W}{g} w \cos\left[\frac{\pi}{2} - \left(\frac{\theta + \phi}{2}\right)\right]$$

$$= \frac{W}{g} \cdot w \sin\left(\frac{\theta + \phi}{2}\right)$$

The only difficulty in giving a value to this expression is in finding w. If the chord of the blade is l feet long and b is the height of the blade inclined ψ to the tangent at entrance, then $bl \cos\psi$ is the aspect of the blade in the direction of the tangent at entrance and

$$W = \cdot 08 \times \sqrt{u^2 + v^2} \cdot bl \cos\psi$$

and the force producing rotation is then

$$F = \frac{\cdot 0024\, bl(u^2 + v^2)\cos\psi \cdot \sin(\theta - \phi)\sin\left(\frac{\theta + \phi}{2}\right)}{\cos\left(\frac{\theta - \phi}{2}\right)} \qquad (7)$$

and the work done is Fu. (8)

The kinetic energy of the air is $\frac{Ww^2}{2g}$, so that the efficiency may be written as

$$\frac{2Fug}{Ww^2} \qquad (9)$$

According to Molesworth (*Pocket Book*, 25th Edn., p. 552), for wooden mills the horse-power

$$H = 0\cdot00000091\ AV^3$$

where A is the total area of all the sails,

V the velocity of wind, feet per second.

[The velocity of the vane-tips is also given by this authority as 2·6 V.]

The general principle of this rule is easily understood. The total pressure normal to all the blades will be proportional to AV^2, and the component of this in the plane of the wheel is also proportional to AV^2. If the speed (of rotation) of the centre of pressure on any blade be U, then the work done varies as AUV^2; and since U will bear some ratio to V, the work may be regarded as proportional to AV^3.

This authority gives as the suitable angle for the sail with the plane of motion

$$A = 23^\circ - 18\left(\frac{D}{R}\right)^2 \qquad (10)$$

where D is the distance from the centre

R is the overall radius.

In the case of wooden mills there must also be taken into consideration the inclination of the axis of rotation (8° to 15° from the horizontal). This will scarcely affect the intensity of the wind pressure, but will increase the frictional losses at the main bearing.

The application of the above rules concerning the efficiency and forces in each blade necessarily becomes rather complex in designing.

Thus in dealing with plane blades and considering the whole blade we have to remember that A, V_o and ψ are all functions of the radius.

$$\delta A = \beta r . dr$$

where β is the angle of the sector bounding the blade (in diverging blades) and r the radius,

$$V_o = \sqrt{r^2 \omega^2 + v^2} \qquad \omega = \text{angular velocity.}$$
$$\psi = \tan^{-1}\left(\frac{r\,\omega}{v}\right)$$

Putting these values in (3) we have as the torque

$$\int_{r_1}^{r_2} \frac{\cdot 0016 . \beta r^2 . \sqrt{r^2 \omega^2 + v^2} . \left[\theta - \tan^{-1}\left(\frac{r\,\omega}{v}\right)\right] . \cos\theta . dr}{30^\circ}$$

where r_2 and r_1 are the external and internal radii.

Substituting radian values for the angles and taking constants outside.

$$\text{Torque} = \frac{\cdot 0096 \times \beta . \cos\theta}{\pi}$$
$$\int_{r_1}^{r_2} r^2 \sqrt{r^2 \omega^2 + v^2} . \left[\theta - \tan^{-1}\left(\frac{r\,\omega}{v}\right)\right] . dr$$

If the function under the integral sign is plotted as a function of r, and the area between r_1 and r_2 integrated with a planimeter, and the result multiplied by the constant factor outside, the result is the torque on each blade.

$$M \times \text{torque} \times \omega = \text{work done per second.}$$

[M = number of blades.]

CHAPTER VII

TRAIN AND MOTOR RESISTANCE

In connection with vehicles, wind pressure is important in two respects:

(1) As a force-retarding motion, the wind being generally mainly due to the vehicle's own motion.

(2) As an overturning force.

A vehicle running at a speed of u miles per hour against a wind with a component velocity in the line of the vehicle's motion v, is subject to a resistance which varies as $(u + v)^2$. $(u + v)$ here corresponds to V in our previous problems, being the relative velocity of the wind to the vehicle. If the wind blows in the same direction as the vehicle is moving $V = (u - v)$.

The resistance will depend on the various items given in chap. ii. Generally, railway locomotives and motor-cars have a roughly square front, but the length is so great that suction is small. On the other hand, there are generally a number of surfaces other than the front one more or less exposed to the wind, so that

unless special care is taken in formation of the front of the vehicle

$$R = \cdot 0025\ A\ V^2 \text{ (Harding's formula)}$$

where R is the resistance in lbs.

A the frontage area in square feet.

V the velocity in *miles per hour.*

[*Note.*—This is the only case in which it is desirable to measure V in miles per hour:

60 miles per hour = 88 feet per second.]

It is very probable that the actual resistance at high speeds is more than this, but in view of the somewhat uncertain value of the road resistance, very accurate figures have not yet been found.

A locomotive with the cab leading will necessarily have greater resistance than one with the firebox leading, since the stream lines will diverge and probably clear the cab altogether in the latter case.

Similarly, a motor-car with a wind-guard of the usual sheet form will have a much greater resistance than without.

The passage of the air under the vehicle over irregular projections (axles, springs, brake mechanism, &c.) will greatly increase the resistance, and turbulence will occur in this region with consequent creation of dust. If a sheet (preferably metal) of smooth surface and outline be fixed under the vehicle this turbulence and

resistance (except in the immediate neighbourhood of the wheels) will be greatly decreased.

Similarly, by forming the front of the car or locomotive with a prow of stream line form, the resistance may be enormously decreased. A vehicle with open top or windows will be more resisting than one completely enclosed, and gaps between a train of carriages will increase the resistance in proportion to the size of the gap. Hence corridor trains are generally of less resistance than those with separate carriages.

As regards the resistance of wheels, whether connected or not to vehicles, an approximate idea of the energy consumed may be computed as follows:

Disc Wheels.

These may be considered as having three sources of resistance, (1) rim and (2 and 3) the sides.

If the overall diameter be D the surface of the rim is π DB where B is the breadth. If the revolutions per second are n, then πDn is the surface velocity and the resistance is

$$r_1 = \gamma \cdot \pi \, DB \cdot (\pi \, D \, n)^2$$
$$= \gamma \, \pi^3 \, D^3 \, B \, n^2$$

γ here is the coefficient of skin friction resistance (= about ·000005 lb. per square foot at unit velocity).

As regards the sides, the area of any concentric

element is $2\pi r.dr$, where r is the radius and the velocity is $2\pi rn$, so that the resistance for both sides is

$$r_2 = \gamma \,.\, 16\,\pi^3 n^2 \int_0^R r^3 .\, dr$$

$$= \frac{\gamma \,.\, 16\,\pi^3 n^2 R^4}{4} = \frac{\gamma\,\pi^3 n^2 D^4}{4}$$

where R is the external radius $\left(= \frac{D}{2}\right)$ and the total resistance is then

$$\gamma\,\pi^3 D^3 n^2 \left(B + \frac{D}{4}\right) \qquad (1)$$

Spoked Wheels.

The resistance of this type is much greater. The rim may be calculated as for a disc wheel and the boss may be neglected.

If there are N spokes, B thick (*i.e.*, transverse to the plane of the wheel), then the exposed area of any element at radius r is N B.dr and the speed as before $2\pi rn$, so that the resistance of the element is

$$K \,.\, 4\,\pi^2 N\, B\, n^2 \int_0^R r^2 \,.\, dr$$

$$= \frac{4\,K\,\pi^2 N\, B\, n^2 R^3}{3} \qquad (2)$$

K is here ·0016, as in previous instances of direct resistance. The fact of the spokes being rounded may be disregarded as there is very considerable turbulence.

With respect to the second consideration, overturning effect, it is obvious that, by the principle of moments, when the vehicle is about to turn over

$$Ph = \frac{W}{2} . D$$

where P is the total wind pressure,

h is the height of the centre of pressure,

W is the weight of the vehicle,

D is the gauge of the track (*i.e.*, diameter of wheel base).

Writing $P = KAV^2$ where A is the exposed area of the side of the vehicle, K and V^2 are as before, and calling $h = \frac{1}{2}(H - h_0)$ where h_0 is the height from the track to the underside of the vehicle, and H is the height to the top of the vehicle

$$KAV^2 . \tfrac{1}{2}(H - h_0) = \frac{W}{2} D$$

so that the limiting value of V^2

$$= \frac{W D}{K A (H - h_0)} \qquad (3)$$

On curves this must be combined with the centrifugal force, with which it might under some circumstances co-operate.

CHAPTER VIII

EFFECT OF WIND ON WATER

THE effect of wind on rivers and open seas may be classified into two types.

(1) Wave-making.

(2) Acceleration of the whole mass.

The subject of wave propagation is not very fully understood, but an excellent *résumé* of the existing information on the subject is given in Sir W. White's *Text-Book of Naval Architecture* (pp. 216, *et sqq*).

The principal researches on this subject have been carried out by French experimenters and observers, especially Admiral Coupvent Desbois, Lieutenant Paris, and M. Antoine.

[*Comptes-rendus*, 1866, *Revue Maritimé*, vol. xxxi., and *Des Lames de haute mer*, 1879.]

It is certain that the continuous action of the wind in one direction will generate a wave in that direction, but the form of the ocean bed and internal friction greatly modify the formation of the waves. Waves

once formed tend to be increased by the wind on account of the lateral surface exposed. There is a limiting velocity for the waves, beyond which the wind produces no effect other than in maintaining the perturbation. If the wind dies away, the period and length of the waves remains practically constant, but the amplitude decreases, being finally damped out by friction.

There is therefore great difficulty in estimating the relation between wind and wave.

The following hypotheses have been suggested.

(1) Cube of amplitude of waves and square of speed of wind. (*Coupvent Desbois.*)

(2) Speed of wind = ·022 (speed of wave)^2 (Feet-seconds unit). (*Lieutenant Paris.*)

(3) The following formulæ are given by Sir W. White, based on M. Antoine's work and Admiral Desbois' law:

$$2H = 0{\cdot}75\, v$$
$$2L = 30\, v^{\frac{1}{2}}$$
$$2T = 4{\cdot}4\, v^{\frac{1}{4}}$$
$$V = 6{\cdot}9\, v^{\frac{1}{4}}$$

Here 2L = length of wave (metres).

2T = period of wave (seconds).

2H = double amplitude (metres).

V = velocity of waves (metres per sec.).

v = " wind (" ").

Beyond this there is practically no information and persons interested cannot do better than study the tables given in Sir W. White's book.

Passing to the consideration of the effect of the wind on the velocity of flow of masses of water, it is first necessary to point out that the velocity of flow in a mass of fluid is at a maximum at points roughly midway between the sides of the channel, and a little below the surface. The most exact information on this subject is that collected by various hydrographers for the U.S.A. Government in connection with the Mississippi and other rivers.

If a curve be plotted showing the velocity as a function of the depth, the form will be found to be approximately parabolic.

In the absence of the wind the vertex of this parabola is about ·3 of the depth from the surface. A downstream wind causes the vertex to rise, and an upstream wind causes it to sink. The vertex never rises quite to the surface. According to the Mississippi experiments, if the force of the wind be graduated from calm to hurricane as 0 to 10 and h^1 is the depth of the axis of the parabola, m the hydraulic mean depth (sectional area ÷ wetted perimeter)

$$\frac{h^1}{m} = 0 \cdot 317 \pm 0{\cdot}06 f$$

[See Unwin "Hydromechanics" *Encyclop. Britt.*: Edn. ix.] Thus in the case of a wind graduated at $\frac{4}{10}$ of a hurricane (pressure about 20 lbs. per square foot) downstream increased the maximum velocity from 7·92 units to 8·1 and raised the vertex to $\frac{1}{10}$ of the depth from the surface. The same wind upstream decreased the maximum velocity from 7·92 to 7·88 and lowered the vertex to half the depth.

It is possible to formulate an equation to this parabola (as has been done by M. Boileau), but in view of the uncertain nature of the circumstances in any concrete example no useful purpose woud be served.

CHAPTER IX

SCOURING EFFECT OF WIND

The effect of the dust-laden wind in rubbing away stone and other hard substances is well known, and may be compared to the method of excavating with high velocity jets. The erosion of land has been shown by geologists to be partly due to this, and a practical application of the principle has been applied in the process of sand-blasting glass.

The air always contains many minute particles, these being, however, generally very light and soft; when, however, it is in rapid motion heavier particles are picked up and carried along. The exact dynamical character of this process is not completely understood, but it would appear that a single particle resists the passing air with a force depending on its form and the velocity of the wind. If this force exceeds that of the friction and adhesion between it and the material in which it rests, acceleration will necessarily be given to it, and by its kinetic energy it will travel in a direction

only slightly inclined to the direction of the wind. The reason for the smallness of the inclination lies in the small value of the weight as compared with the accelerating force.

Such a particle will eventually acquire almost the average velocity of the wind, and striking on any surface will exert a force depending on its momentum and the manner and time in which such momentum is altered.

A rough approximation to the force exerted may be obtained by considering the whole mass of air, which travels with the particles as being, in virtue of their presence, so much heavier, so that the momentum per second is

$$\frac{W v}{g}$$

where W is the weight of a cubic foot of the air and sand, v is the mean velocity, and $g=32 \cdot 2$. In air alone, as we have seen, $\frac{W}{g} = \cdot 0024$ (about), so that when sand is present $\frac{W}{g} > \cdot 0024$, say,

$$\frac{W}{g} = c \times \cdot 0024 \qquad (1)$$

where $c > 1$.

The value of C is quite uncertain, but it would seem reasonable to suppose it may reach values of 2 or 3, so that the force of the sand-blast may be 2 or 3 times

as great as that of the wind, other things being the same.

We must, however, be careful not to confuse this direct acting force with the scouring force. The latter is almost wholly due to the friction between the fluid and the surface.

It is difficult to assign any exact value to this, and in the absence of any better hypothesis the author suggests that the usual principle of assuming viscosity to vary as density will apply. Thus in the case of water

$$R = f A V^2$$

where f is a coefficient about ·003 to ·007,

A is area of wetted surface in square feet,

V is velocity (feet per second)^2,

R is resistance in lbs.

This is reduced in the ratio of the density of air to water (about $\frac{1}{800}$), so that for air we write, as already mentioned,

$$R = \cdot 000005 \ A V^2$$

If now, to take into account the sand or other suspended matter, we multiply by the above-mentioned constant c, the formula will express the value of the resistance as nearly as the empirical rules will allow. The actual effect of a dust-laden wind of a given

density and velocity will, of course, depend on the physical structure of the surface scoured.

In the case of natural denudation the driving of the particles by the wind is probably the principal cause of the attrition.

Numerous other instances might be adduced in which wind pressure plays an important part, but the author has endeavoured to confine his attention to the cases which affect engineering practice only.

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